First Approach for the Modelling of the Electrical Field surrounding a Piezoelectric Transformer in view of Plasma Generation

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Abstract — This paper introduces an open multi-physics modeling problem issued from recent investigations in application of piezoelectric transformers for the plasma generation. The electrical field distribution surrounding the transformer is preliminary studied according to a weak coupling formulation. This numerical modeling relies on a Finite Difference Method (FDM).

I. INTRODUCTION

The piezoelectric transformers are commonly used in widespread of applications. Recently, several studies have underlined the ability to exploit the electrical potential on piezoelectric element undergoing mechanical vibrations to generate non-equilibrium plasma [1].

In plasma generation applications, the piezoelectric transformer is deviated from its common two-port network for an exploitation of the high electrical potential on the surface of the secondary part. Generally, a classical Rosen type transformer is used for its high voltage step-up ratio. It consists of a single rectangular piece of piezoelectric ceramic material: the primary part is poled in the thickness direction whereas the secondary part is poled in the length direction. This architecture is driven by an AC voltage power supply applied to the driving part at frequency close to a length extensional mode resonance frequency. Experimentally it can be shown that plasma is generated at the secondary part of the transformer [1]. This plasma is due to the surrounding electrical field issued from the electric potential distribution on the secondary part of the transformer. Thus, in order to understand how the plasma is generated, one must first study the distribution of the surrounding electric field.

The distribution of the electrical potential produced on surface of the secondary part is obtained from a previous analytical modelling [2]. From this analytical result, the distribution of the surrounding electrical field is computed by means of a numerical model. The objective of this numerical modelling is to show that this first approach allows understanding some experimental observations.

II. ELECTRO QUASI STATIC FIELD APPROXIMATIONS

The electric potential distribution on the surface of the secondary part of the transformer creates a time varying electric field. In this first approach, we study the surrounding electric field at atmospheric pressure so there is no plasma generation.

The equations of electromagnetic field surrounding the transformer are as follow:

$$\mathrm{curl}\,\mathbf{E} = -\partial\,\mathbf{B}/\partial t \quad (1) \qquad \mathrm{div}\,\mathbf{D} = 0 \quad (3) \qquad \mathbf{D} = \varepsilon_0\,\mathbf{E} \quad (5)$$

$$\mathrm{curl}\,\mathbf{H} = \partial\,\mathbf{D}/\partial t \quad (2) \qquad \mathrm{div}\,\mathbf{B} = 0 \quad (4) \qquad \mathbf{B} = \mu_0\,\mathbf{H} \quad (6)$$

where **H** is the magnetic field, **B** is the magnetic flux density, **E** is the electric field, **D** is the electric flux density, μ_0 is the magnetic permeability and ε_0 is the electric permittivity.

Generally, in low frequency applications, one refers to the relaxation time which depends on the electric permittivity and the conductivity of the medium to neglect displacement current. A Magneto Quasi Static (MQS) field approximation can then be done and the displacement current can be neglected.

But here, the conductivity of free space is very low and displacement current can not be neglected. Equation (2) means that a magnetic field is created by the displacement and equation (1) that this magnetic field may interact in return with the electric field. Equations (1) to (6) leads to the electromagnetic wave equation:

$$\mathrm{curl}\,(\mathrm{curl}\,\mathbf{E}) + \varepsilon_0\,\mu_0\,\partial^2\,\mathbf{E}/\partial t^2 = 0 \qquad (7)$$

For time harmonic electromagnetic field, this wave equation may be associated to the characteristic number:

$$(1/L)^2 - (\omega/c)^2 \qquad (8)$$

where L is the typical length scale of the problem, c the speed of light and ω the angular frequency of the field.

In our problem, the frequency of the AC power supply is less than 300 kHz [2]. So we have:

$$c/\omega > 159\ \mathrm{m} \qquad (9)$$

In this first approach, we consider that this value is very great in front the typical length of a piezoelectric transformer [2]:

$$L < 0.1\ \mathrm{m} \qquad (10)$$

In this case, the wave propagation can be neglected and the problem can be considered as an Electro Quasi-Static (EQS) field problem [3] and in equation (1) $\partial\mathbf{B}/\partial t$ can be neglected.

III. 2D ELECTROSTATIC FIELD PROBLEM

As there is no dissipation in the surrounding air, the EQS problem can be considered as a succession of static field problem: at each time the electric scalar potential is computed by solving a Laplace's equation where the electric scalar potential is imposed on the surface of the secondary part of the transformer. For a given mode of the transformer, the electric potential on the points of this surface are sinusoidal function of time and are all in phase but with different magnitude depending on location points on the surface. So, for a given mode, only one solution of the Laplace's equation is needed to know the distribution in space and the evolution in time of the electric field. Furthermore, previous studies have shown that the magnitude of electric potential on the surface can be assumed to vary only in one direction [2]. The problem can then be reduced, in this first approach, to a 2D electrostatic field problem.

IV. NUMERICAL MODEL OF THE ELECTRIC FIELD

In order to solve the steady state problem of electrical potential surrounding the piezoelectric device, the Finite Difference Method (FDM) is applied because it can be easily implemented for this first approach [4]. Fig.1 shows the rectangular study domain *D* in which the upper-half part of a Rosen type PT is considered. Ω, Ω_{in}, Ω_{out} and Γ_D represented respectively the surrounding medium, the primary and secondary sides and the Dirichlet boundary conditions (i.e. the electrical potential is vanished on Γ_D).

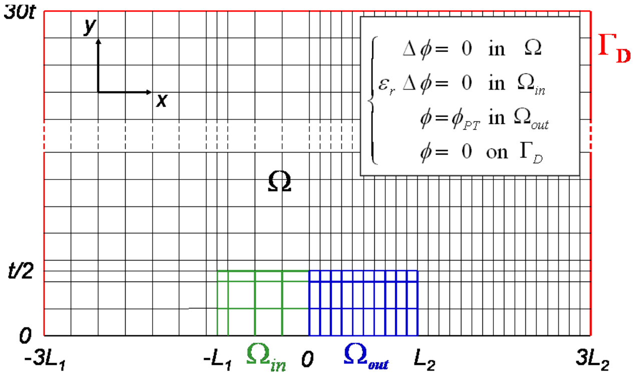

Fig.1 Domain definition of the numerical problem of the electrical potential generated by a Rosen type PT

The Rosen type PT is supposed to be surrounded by a free charges air of vacuum permittivity ε_0. The driving part is considered as a dielectric medium with a high relative permittivity (≈1000). A z-invariance and an x-axis symmetry are supposed.

The electrical potential produced by a Rosen type PT follows consequently the boundary problem reminded in Fig.1, where Δ is the Laplace operator and ϕ_{PT} is the electrical potential from the analytical model [2]. Thus, the electrical potential follows the Laplace's equation as well in the primary side as in the surrounding medium. In order to discretize this elliptic PDE, the 5-point stencil is used [4]. It is clear that this model is a weak coupling involving an analytic model for the equations of the piezoelectricity in the transformer and a numerical 2D-model for the electric field equation.

V. RESULTS

The numerical 2D-model is simulated for the first four extensional modes of a Rosen type transformer. As an illustration, a cross-section view obtained for the third longitudinal vibratory mode (i.e. 3/2λ-mode) is shown on Fig.2(a). A front view of the experimentally observed glow discharge is also compared Fig.2(b). This phenomenon was generated in air under a pressure of 2.4 mbar with a CMT PT sample manufactured by the Noliac Group supplied with a sinusoidal voltage of an amplitude of 1.8V and a frequency of 207.6 kHz. It can be noted the well agreement between the high electrical potential areas and the luminescent areas. Moreover, the other numerical results (λ/2- and λ-modes) also corroborate the experimental observations. The most notable fact is the lack of glow discharge for the 2λ-mode because of the weak electric field emphasized by the numerical study.

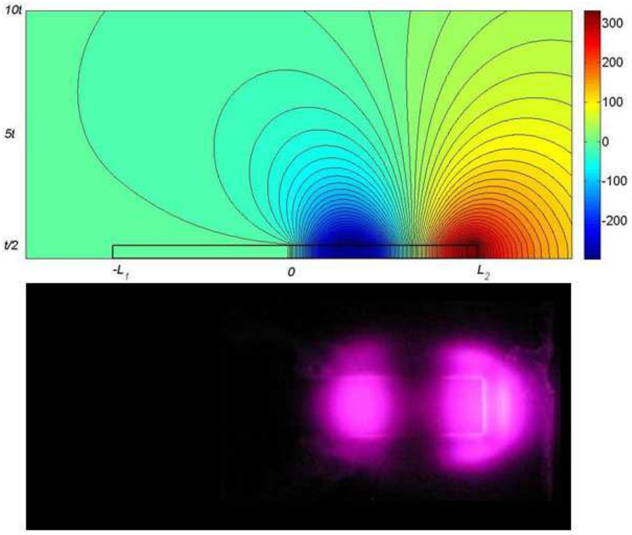

Fig. 2. (a) Numerical electrical potential distribution (kV) in cross-section view (3/2λ-mode) (b) Experimental glow discharge in front view (3/2λ-mode)

VI. CONCLUSION

With the objective to move the piezoelectric and plasma domains closer, this preliminary study has concerned the weak coupling of piezoelectricity and the surrounding electric field. Results obtained are very interesting and encouraging.

In light of experimental observations, which have shown a significant influence of glow discharge on the electromechanical behaviour of the piezoelectric transformer, it clearly appears that the coupling relation is stronger than suggested in this paper. The modelling of this open multi-physics problem will require a more accurate study linking the electric field equations with the local equations of the piezoelectricity.

VII. REFERENCES

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